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Dynamics of α-clusters in N = Z nuclei

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Abstract. The systematics for binding energies per α-particle in $N = Z$ nuclei, $E_{B\alpha}/N_\alpha$, are studied up to ^{164}Pb. It is shown that, although a geometrical model can be used to explain the systematics for light nuclei, the binding energy per α-particle exhibits structures which are due to the well-known shells of the mean field of nucleons in nuclei. The overall dependence of $E_{B\alpha}/N_\alpha$ on N_α in $N = Z$ nuclei (for the ground-state masses) can be described in a liquid-drop model of α-particles. Conditions for a phase change with the formation of an α-particle condensate, a dilute Bose gas in excited compound nuclei are discussed for $E_{B\alpha}/N_\alpha = 0$, at the thresholds. This is achieved when the binding energy per nucleon in nuclei is equal to or smaller than in the α-cluster. At somewhat smaller excitation energies the appearance of a Bose gas with a closed-shell core ($N = Z$, *e.g.* of ^{40}Ca) is proposed within the same concept. The experimental observation of the decay of such condensed α-particle states is proposed with the coherent emission of several correlated α-particles not described by the Hauser-Feshbach approach for compound-nucleus decay. This decay will be observed by the emission of unbound resonances in the form of ^{8}Be and ^{12}C$^*(0_2^+)$ clusters.

PACS. 21.10.-k Properties of nuclei; nuclear energy levels – 21.60.Gx Cluster models

1 Introduction

The binding energies of nuclei as a function of mass number show a peculiar systematic behaviour, which often is discussed to be related to the formation of α-clusters. In fact, the specific dependence of the nucleon-nucleon force on the spin and isospin quantum numbers [1], both coupled to zero, produces very strong binding in the spin- and isospin-saturated α-particle substructures. Its high binding energy and the internal structure and symmetry give a 30% higher density than the usual central density in nuclei. The α-particle is the unique cluster subsystem in nuclei.

This feature is well known from the early history of nuclear science, but it is also borne out in the most recent model-independent calculations of nuclear structure, like in the anti-symmetrised molecular dynamics (AMD) calculations of Horiuchi and Kanada-Enyo [2–5], and in a related approach by Feldmeier *et al.* [6,7]. In these calculations the density distributions of the nucleons are obtained and the ground states of light nuclei already show strong clustering effects. Even more spectacular are the results for loosely bound nuclear systems [4,5], where α-clusters appear naturally as dominant substructures. This work has established that α-clusters have a decisive role in the description of light nuclei, in particular for the loosely bound neutron-rich isotopes. The extra neutrons are found in molecular orbitals of two α-particles forming a bound molecular two-centre system for the beryllium isotopes [8].

Furthermore, the α-particle is the important ingredient in the concept of the Ikeda diagram [9–11], where highly clustered states (*e.g.*, linear chains) are predicted at excitation energies around the energy thresholds for the decomposition into specific, constituent cluster channels.

In the present work the dynamics of α-clustering in excited states of heavier $N = Z$ nuclei will be explored. For this purpose the systematics of binding energies per α-particle in nuclei $E_{B\alpha}/N_\alpha$ are reviewed in sect. 2. In sect. 2.2 a liquid-drop model (LD) based on α-particles for $N = Z$ nuclei is explored. Some concepts needed to discuss dilute (super-fluid) multi-α-cluster systems, a Bose gas, are exemplified in sect. 3. In sect. 4 some experiments for the formation of nuclei in states with an α-particle gas are proposed. This Bose gas of α-particles has the properties of a condensate because of the very large de Broglie wavelength of the α-particles and their coherent properties. The observation of α-particle super-fluidity in the decay of compound nuclei at the appropriate excitation energy is proposed via the coherent and enhanced emission of mul-

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tiple α-particles in the form of ^{8}Be and ^{12}C$^*(0_2^+)$ clusters (see also [12,13]).

2 Binding energy of α-particles in N = Z nuclei

2.1 Binding between α-particles: a historical overview

Historically, the first models of the nucleus before the discovery of the neutron were based on α-particles [14, 15]. Of interest for our study is the work of Hafstad and Teller [16]. There it was noticed that the binding energy of the α-particles in light $N = Z$ nuclei show a linear dependence, if plotted as a function of the number of bonds, which can be deduced by counting the touching points in a geometrical model. The number of bonds is obtained by placing the α-particles in a close-packing arrangement, as done for atoms in a cubic-centred arrangement in solid-state physics. Later, α-particles as a basic structure for $N = Z$ nuclei have been used extensively in nuclear models. As the most successful approach we can consider the Bloch-Brink–α-cluster model [17–19]. In the AMD approach the ground-state binding energies, of *e.g.* ^{12}C and ^{16}O, are well reproduced in a basis using α-particles, if, in addition, a mixture with shell model configurations is introduced [20]. In this work the role of the spin-orbit force in breaking the α-cluster structure in the ground states is illustrated.

We follow the concept to calculate the nuclear binding energy per bond using a geometrical model for $N = Z$ nuclei, with the close packing of rigid spheres. The value due to the nuclear binding per bond has to be obtained by subtracting the Coulomb energy from the total binding energy. The binding energy per α-particle bond is obtained by using the total binding energy (E_B^t) from the mass tables and calculating the Coulomb energy, E_c^t, using the liquid-drop formula with $E_c^t = -0.715Z^2/A^{1/3}$. Thus, the intrinsic nuclear binding energy divided by the number of bonds, N_b, between the α-particles is obtained,

$$E_{N_b} = \frac{[E_B^t - E_c^t - E_B^\alpha N_\alpha]}{N_b}\,, \tag{1}$$

where E_B^α (= 28.3 MeV) is the binding energy of the α-particle and N_α is the number of α-particles. The number of bonds is determined from the number of points, at which, for a geometrical model, spheres representing α-particles, would touch. This number can vary between particular geometrical arrangements, because not only spherical shapes appear (like in the case of ^{28}Si). We find with some small variations that the binding energy per bond is around 5 MeV in light nuclei, and approximately 6.3 MeV in medium mass and heavier nuclei, as shown in table 1. Counting the number of bonds works well for the light-mass nuclei, ^{16}O to ^{40}Ca, and has been tried also for heavy nuclei like ^{100}Sn or even ^{164}Pb. However, the number of bonds for nuclei heavier than ^{52}Fe can only be estimated and there is often more than one way of summing the bonds, which are given in parentheses in table 1. The masses have been taken from ref. [21], for ^{164}Pb from a theoretical study of the $A = 164$ region [22].

Table 1. The nuclear binding energy per bond, E_{N_b}, for α-particles in close geometrical packing in $N = Z$ nuclei; N_α —number of α-particles, N_b —number of α-α bonds, E_B^t —total binding energy, E_c^t —Coulomb energy; all energies are quoted in MeV.

Nuclide	N_α, N_b	E_B^t	$E_B^t - E_c^t$	E_{N_b}
^{4}He	1, 0	28.3	–	–
^{12}C	3, 3	92.16	103.45	6.18
^{16}O	4, 6	127.6	145.8	5.43
^{20}Ne	5, 9	160.7	186.9	5.04
^{24}Mg	6, 12	197.2	233.9	5.34
^{28}Si	7, 15	236.5	282.7	5.64
^{32}S	8, 18	271.8	332.1	5.87
^{36}Ar	9, 21	306.7	376.9	5.82
^{40}Ca	10, 24	342.0	425.8	5.95
^{52}Fe	13, 33	447.7	577.3	6.34
^{56}Ni	14, 36(37)	483.9	630.6	6.5(6.33)
^{72}Kr	18, 48(52)	607.1	832.0	6.4(5.92)
^{80}Zr	20, 54(60)	669.8	935.7	6.8(6.16)
^{100}Sn	25, (80)	824.5	1210.1	(6.28)
^{164}Pb	41, (130)	1200.1	2079.9	(7.07)

Counting the number of bonds may lose its meaning for the heavier nuclei, because geometrical considerations are less valid, and many configurations of "close packing" become possible. However, in an α-particle liquid consisting of spheres, the touching points and the binding effects for the spheres inside the configuration should correspond to a saturation value. Therefore, a liquid-drop model based on α-particles for the heavier $N = Z$ nuclei can be discussed.

Of further interest is the total binding energy of all α-particles in an $N = Z$ nucleus. For this purpose the binding energy per α-particle, $E_{B\alpha}/N_\alpha$, is calculated (fig. 1),

$$E_{B\alpha}/N_\alpha = [E_B^t(N,Z) - N_\alpha E_B^\alpha]/N_\alpha. \tag{2}$$

The same quantity has also been calculated for a core of ^{40}Ca (or ^{52}Fe), with the number of α-particles outside the core being defined as $N_\alpha^{'}$, see fig. 1:

$$E_{B\alpha}^{^{40}Ca}(N_\alpha^{'}) = \\ \left[E_B^t(N,Z) - E_B^{^{40}Ca}(N,Z) - (N_\alpha - 10)E_B^\alpha\right]/(N_\alpha - 10). \tag{3}$$

The excitation energies where this value approaches zero —a point where an α-particle condensate can form— will be discussed below, see sect. 3.

Here the systematics of the binding energies of $N = Z$ nuclei, as compiled by D.H.E. Gross [23], may be mentioned, where α-particle substructures are also invoked. Contrary to the claim made in ref. [23], the systematics of binding energies (of, *e.g.*, 2p or 2n pairs) of α-particles, show irregularities for "magic" numbers which, however, fail to fit to "magic" numbers of the close-packing configurations, like $N = 7, 13, 19$, cited in general for van der Waals "cluster physics" (see, for example, ref. [24]).

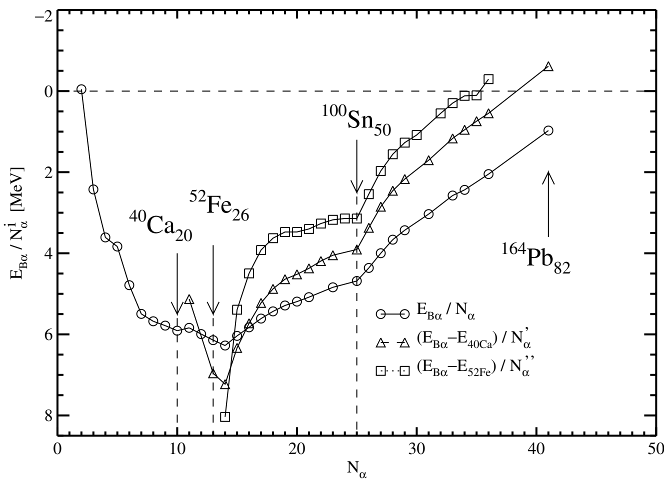

Fig. 1. The binding energy per α-particle in $N = Z$ nuclei. The lines are drawn to connect the points. The same quantities are shown under the assumption of two different heavy clusters as cores: ^{40}Ca and ^{52}Fe as indicated.

This failure is decisive. The irregularities appear at the "magic" numbers which are well known in nuclear physics and describe features of the mean-field shell model of nucleons. The numbers in both models are unique and a deviation by one unit is enough to rule out a model. For the close packing of α-particles in the ground states of nuclei the anti-symmetrisation of the nucleons and appropriate symmetry transformations produce wave functions which are equivalent to a shell model basis. This point has been discussed repeatedly. Therefore, in close packing of α-particles which are mutually overlapping, eventually the magic numbers of the nuclear shell model must prevail.

2.2 Liquid-drop model for N = Z nuclei with α-particles

We shortly discuss a liquid-drop model (LDM) of nuclei consisting of α-particles. Here only an approximate expression which describes the macroscopic trend of the binding energies, as used in textbooks [25], is of interest. The aim is to test the validity of the concept of bonds between α-particles in a liquid.

The binding energy of $N = Z$ nuclei is expressed as a function of the number of α-particles equivalent to the standard LDM [25]. The expression will be functionally the same as for the LDM for nucleons without the asymmetry term:

$$E^t_B = a_V(\alpha) N_\alpha - a_{Sur}(\alpha)(N_\alpha)^{2/3} + E^t_c\,, \tag{4}$$

where $a_V(\alpha)$ and $a_{Sur}(\alpha)$ are the coefficients for the volume and surface energies, respectively. Dividing by the number of α-particles, N_α, the general trend, as shown in fig. 1, should be obtained. A fit to the points will be only approximate and needs a more complicated functional dependence to describe the whole range of masses. The values of the constants which are obtained from an approximate fit to the total binding energies are

$$a_V(\alpha) = 62\,\text{MeV} \tag{5}$$

and

$$a_{Sur}(\alpha) = 42.1\,\text{MeV}. \tag{6}$$

For the interpretation of these values, the rescaling of the coefficients in the LD mass formula needs the replacement of the total number of nucleons (A) by ($A/4$). This gives 15.5 MeV for the volume coefficient, a_V. This value is very close to the standard value ($a_V = 15.6$ MeV) of the LD model for nucleons used in the textbooks like ref. [25], this is in fact a trivial agreement. Similarly for the surface term, $a_{Sur}(\alpha)$, the coefficient with the factor $(A/4)^{2/3}$ can be calculated, which gives the coefficient 16.7 MeV close, to the cited [25] value of 17.2 MeV.

The saturation value for the volume term can be calculated ($a_V(\alpha) = 62$ MeV): the central α-particle is always surrounded by 12 particles, producing $N_b = 12$. Dividing the cited saturation value with this number we obtain 5.16 MeV for the α-α bond energy. This value is indeed close to the values given in table 1. Take ^{164}Pb, $N_\alpha = 41$. The total surface energy using eq. (4) is 499 MeV, which is divided by 5.16 MeV to obtain the number of free surface bonds $N_b = 97$. To complete the result for ^{164}Pb, for the volume we have $12 \times 41 = 492$ bonds and approximately 97 free bonds for the energy to be subtracted (unsaturated bonds at the surface, see also table 2).

Similar values are obtained when we apply the model to ^{52}Fe, the most symmetric and compact cluster configuration with 13 α-particles. The outer α-particles have 7 missing bonds each (in this case it is easy to count the bonds to obtain the surface-energy term). With the total nuclear binding given in table 1, the energy per bond is calculated as 6.21 MeV in close agreement with the value given in table 1.

In conclusion, it may be stated that the concept of bonds in a liquid of α-particles seems to apply to the ground-state masses of nuclei in a reasonable way. The ground states of nuclei are rather well described by a densely packed system of α-clusters (a liquid), which are partially dissolved due to anti-symmetrisation and to the spin-orbit interaction [20]. Actually, in the study of ref. [26] it is shown that in nuclear matter at high density the α-particle phase is destroyed, whereas the pairing correlations survive. We may conclude that the α-clusters with a changed internal structure inside nuclear matter coexist with the fermion gas consisting of nucleons, and the values of the cited bond energies represent some effective value.

3 Formation of condensates

From fig. 1 we can deduce the value for the excitation energy where the nucleus will completely decay into α-particles by multiplying the values of $E_{B\alpha}/N_\alpha$ in fig. 1 with N_α. At this energy the formation of an α-particle

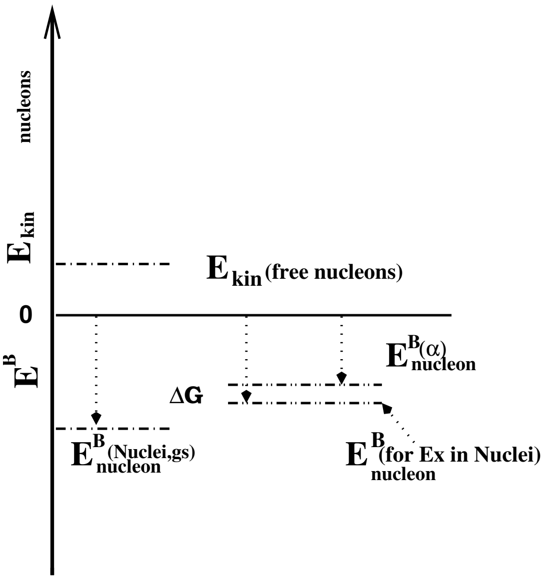

Fig. 2. Schematic illustration of the values of energies of free nucleons and alternatively their binding energies in nuclei, which is generally larger than in α-clusters (7.07 MeV). The difference ΔG between the binding energies decreases with excitation energy in nuclei, at a critical value the two binding energies become equal, a collective state with an α-particle gas can be formed.

condensate is expected. In the ground states of nuclei the intrinsic structure of α-clusters are certainly very different from that of free α-particles, a fact widely discussed in the literature.

We want to discuss the formation of a free α-particle gas, where the average distance between α-particles is much larger and the corresponding nucleon density will be well below normal nuclear densities. In fact, in a theoretical investigation of Bose-Einstein condensates in nuclei by Tohsaki, Schuck *et al.* [27–29], they find that close to the thresholds for multi α-particle decays, the states with α-clusters have a much larger radial extension than the ground states. From the view point of the fermion gas the appearance of such states will depend on the temperature (*i.e.*, excitation energy, E_x^*) of the nucleus. The concept of phase transitions with two components can be used, a concept well established in thermodynamics of composite systems in statistical physics [30].

The basic equation is the "reaction" of four "free" nucleons (two protons and two neutrons coupled to total values of spin and isospin of zero) forming α-clusters: $(N_1+N_2+N_3+N_4) \longleftrightarrow \alpha\text{-particle}+28.3\,\text{MeV}$. The free nucleons, N_i, should have a definite volume and pressure, in order to define thermodynamic quantities and where the density allows the occurrence of the mentioned reaction. This can only be done in a model like AMD [5], where a certain number of nucleons are confined in a volume with a positive kinetic energy, as suggested in fig. 2. In this model a cooling method is applied to find the states of the lowest energy. The energy of the nucleons inside the nucleus is defined by the volume, as the Fermi energy deduced from the nuclear radius in text books [25]. In the AMD approach a certain phase with α-clusters appears, before the formation of the ground states. At the end, the formation of normal nuclei with a binding energy per nucleon higher than in the α-cluster is usually observed. For weakly bound nuclei the α-clusters are obtained in a "natural" way. With the binding energy of nucleons in the α-particle of 7.073 MeV, the nucleons would preferentially form an α-cluster phase in normal nuclei, only with increasing temperature of the nucleus, *e.g.* with increasing excitation energy. This excitation energy becomes rather low in neutron-rich light exotic nuclei (where clustering appears as the dominant structure [5]) and in very heavy $N=Z$ nuclei.

Table 2. Alpha-particle binding and excitation energies for the condensation condition in nuclei with $N=Z$; N_α —number of α-particles, E_B^t/N_n —binding energy per nucleon, $E_{B\alpha}/N_\alpha$ —binding energy per α-particle, E_x^{crit} —condensation energy. The last column shows the values for the case of a ^{40}Ca-cluster core, see also table 1. All energies in MeV.

Nuclide	N_α	E_B^t	E_B^t/N_n	$E_{B\alpha}/N_\alpha$	E_x^{crit}	E_x^{crit}
^{4}He	1	28.3	7.073	–	–	(^{40}Ca)
^{12}C	3	92.16	7.680	2.425	7.27	–
^{16}O	4	127.6	7.976	3.609	14.44	–
^{20}Ne	5	160.7	8.032	3.83	19.17	–
^{24}Mg	6	197.2	8.260	4.787	28.72	–
^{28}Si	7	236.5	8.447	5.495	38.47	–
^{32}S	8	271.8	8.493	5.677	45.41	–
^{36}Ar	9	306.7	8.519	5.78	52.02	–
^{40}Ca	10	342.0	8.551	5.910	59.10	–
^{52}Fe	13	447.7	8.609	6.143	79.86	–
^{56}Ni	14	483.9	8.642	6.275	87.85	–
^{72}Kr	18	607.1	8.432	5.433	97.8	87.78
^{80}Zr	20	669.8	8.371	5.192	103.8	90.38
^{100}Sn	25	824.5	8.244	4.684	117.1	97.65
^{112}Ba	28	894.8	7.99	3.665	102.6	68.79
^{144}Hf	36	1090.9	7.577	2.074	74.6	19.68
^{164}Pb	41	1200.1	7.317	0.973	39.9	−25.21

For the nucleons confined in the nuclear volume we apply the concepts of statistical physics for the reaction $4N$ - α-particle. The rate of the reaction is governed by the free energy, G, and the chemical potentials, μ_α and μ_n. The chemical potentials are defined as $\mu_i=\delta G/\delta N_i$, $i=n,\alpha$. The thermodynamic free energy depends on the number of nucleons, N_n and on N_α, with $G=G(N_n,N_\alpha)$. The change of the free energy becomes

$$\Delta G = \Delta N_\alpha \mu_\alpha + 4\Delta N_n \mu_n. \tag{7}$$

For the complete phase transition a minimum value of the free energy is needed with the condition $\Delta G=0$. In the nuclear medium ΔG is the difference between the binding energy of the nucleon in the free α-particle and in the nuclear medium, as illustrated in fig. 2.

The kinetic energy of the nucleons determines the temperature, T. However, we will use the temperature of the nucleus, t, related to its excitation energy. In the normal case of a mixed system the relative abundance of N_α to

N_n is a function of temperature (in our case excitation energy) and is obtained through the expression

$$\frac{N_\alpha}{(N_n)^4} = K = \exp\left(-\frac{\Delta G(t)}{RT(t)}\right). \quad (8)$$

The constant K is to be determined by experimental observation (the usual coefficient R in statistical physics appears). For the case of negative $\Delta G(t)$, a decrease of the free energy (corresponding to a large value of the constant K) gives a high density of the α-particle reaction products. A positive value corresponds to an energetic disadvantage for the reaction, resulting in a low density of reaction products. In the case of nuclei, the nucleons are embedded in the nuclear medium and are confined in the nuclear potential created by the mean field of all nucleons. The binding energy per nucleon in nuclei is around 8 MeV or more (dependent on the size of the nucleus and its excitation energy). The nucleons are usually more bound in the nuclear medium (ground states of stable nuclei) than in the α-clusters. The relative positions of these various regimes are illustrated in fig. 2. The change in the free energy of the nucleons in the medium is now the difference between the binding energy in the nucleus and in the α-clusters. Actually, because the chemical potential of the nucleons will depend on the excitation energy in the nucleus (on its temperature), we put this dependence in the expression for $\Delta G(t)$.

Alpha-cluster formation is expected if $4E_B^t/N_n$ is less than or equal to the total binding energy of four nucleons in the α-cluster. As the binding energy per nucleon becomes equal or smaller than in the α-particle, a new phase will be formed, a strongly interacting Bose gas. For binding energies of the nucleons close (larger) to that in the α-particle it becomes possible to form a mixed phase of α-cluster states (liquid) and of nucleons.

We summarize that the α-condensation condition is given by $E_B^t/N_n(E_x^{crit}) \geq 7.07$ MeV, which is the binding energy of nucleons in the α-particle and is the same as $\Delta G(t) = 0$. Alternatively, the phase transition will be achieved at excitation energies of the nucleus, E_x^*, corresponding to the thresholds where the all clusters become unbound, the condition being that $E_{B\alpha}(N,Z) = 0$. This is the original concept of the Ikeda diagram. The Ikeda diagram [9] gives a phenomenological condition for the appearance of clustered states (with the inclusion of other clusters like ^{12}C, ^{16}O, etc.) in nuclei. In fig. 2 the relative values of the binding energies are shown for free nucleons and for nuclei, and in α-particles. The binding energy of nucleons (in fig. 2 $E_B^t/N_n = E^B_{nucleon}$) is usually larger than in the α-particle. The condition for the condensation energy is $E_x^{cond} \geq E_x^{crit}$. The values for different nuclei relevant to this concept are given in table 2. We can state that the Ikeda diagram with α-particles can be deduced from a thermodynamic consideration.

Most important for the properties of the α-particle gas is that they do not represent the "ideal" gas, they interact. Two α-particles form as the lowest state, the ground state of ^{8}Be, a resonance at $E_x^* = 92$ keV. We can calculate the de Broglie wavelength $\lambda = h/\sqrt{(2\mu E_x^*)}$ for this case and have $\lambda = 67$ fm (relative motion between two α-particles). If for higher excitation we incorporate the 2^+ at 3.04 MeV the value of λ is still 12.4 fm. Similarly, three α-particles can form the "famous" state just above the three-α-particle threshold in ^{12}C, the 0^+ at 7.654 MeV (288 keV above the threshold of 7.346 MeV). With these values for three α-particles we again get a similarly large de Broglie wavelength of relative motion. Also the third 0_3^+ at 10.3 MeV excitation recently discussed in ref. [31] can participate in the formation of a multi-α-particle correlation, overall we have values for λ of the same order of magnitude as for ^{8}Be. Because of these values the states at the binding energy threshold consisting of α-particles will form coherent super-fluid states. The resonant states in ^{8}Be and ^{12}C act in a similar way as in the residual interaction in formation of the superfluid neutron pairing states, see ref. [1], volume II. The calculations of Tohsaki, Schuck *et al.* based on an α-α potential reproducing the states of ^{8}Be, gives details on such states, which are located just below or above the thresholds for some light nuclei.

Quite interesting conditions appear for states with a boson gas confined in a volume by an additional potential defined by a strongly bound core. We have considered here the nuclei ^{40}Ca and ^{52}Fe as cores, because of their large binding energies per nucleon. The relevant quantities of the binding energy per α-particle for these cases have been given in fig. 1 and the values of the critical excitation energies are also listed in table 2. The last entry in the last column for the ^{40}Ca core (for ^{164}Pb) has a negative sign, indicating that this nucleus, and somewhat lighter nuclei (actually above $Z = 72$), are unstable in their ground states relative to "Coulomb explosion". These nuclei also become unbound for the last proton or α-particle already at smaller Z-values.

4 Formation and coherent decay of multi-α-particle states

For the formation of heavy $N = Z$ compound nuclei, the heaviest combination of stable targets and projectiles is ^{40}Ca + ^{40}Ca giving ^{80}Zr. For even heavier systems we will have to rely on beams of unstable nuclei, one of the best choices appears to be a ^{72}Kr beam, which has a good chance of being produced in the future with usable intensities. The compound nucleus with a ^{40}Ca target will be ^{112}Ba ($Q = -52.54$ MeV). However, an excess of two or four neutrons (with a more intense beam) would most likely not destroy the special states discussed here. The excess neutrons will be placed in quantum orbits around the clusters, for example like in the $^{9-12}$Be isotopes forming molecular states. Because of the fact that the compound nucleus is very far off-stability the reaction Q-value becomes very negative. With an incident energy close to the Coulomb barrier, the final excitation energy can be rather moderate. These compound nuclei will have very small Q-values for the emission of protons or for several α-particles. Heavier compound nuclei may even be unstable to charged-particle emission in their ground states.

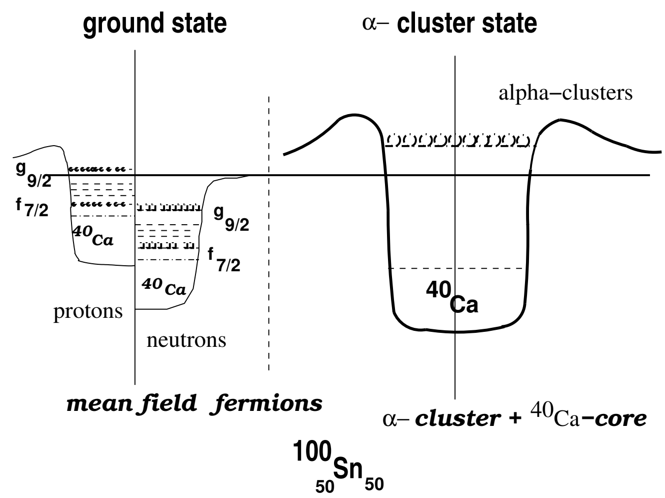

Fig. 3. Schematic illustration of the two models for states in ^{100}Sn. States of low excitation energy, formed by the mean field of nucleons makes the potentials for neutrons and protons rather different due to the Coulomb interaction. Thus, the formation of α-particle structures is strongly suppressed. At the critical excitation energy of 97 MeV (for ^{100}Sn, see fig. 1), a collective state of bosons with α-particles occupying the same orbit outside a ^{40}Ca core will be energetically favoured.

In fig. 3 we illustrate the situation for ^{100}Sn, which can be formed in a reaction with a ^{72}Kr beam and a ^{28}Si target. At excitation energies of 80 MeV or more (for thresholds discussed earlier) many compound nuclear (CN) states will exist consisting of different configurations of the α-particle gas plus a core. Here again the threshold rules apply with respect to excitation energies. We must expect many overlapping states (with a large decay width), which will interact coherently (see ref. [13]). The same compound states can be occupied with a different number of α-particles, these will interact through the 0^+ and 2^+ resonances of ^{8}Be and ^{12}C, depending on the excitation energy of the state. The first experimental observation of such decays with different sharing of α-particles and the resonances can actually be found in refs. [12,13]. The CN states have a large decay width due to α-particle decay channels and many other decay channels.

We are interested in the coherent multiple α-particle emission. Due to the coherent properties of the threshold states consisting of α-particles interacting coherently with a large de Broglie wavelength, the decay of the CN will not follow the Hauser-Feshbach assumption of the statistical model: that all decay steps are statistically independent. After emission of the first α-particle, the residual nucleus contains the phase of the first emission process; the subsequent decays will follow with very short time delays related to nuclear reaction times (or the inverse, decays), favouring the formation of resonances like the $^{12}\mathrm{C}^\star(0_2^+, 2_2^+, 0_3^+)$ states.

Another view for the α-gas is the concept of a collective super-fluid state with a broken symmetry, the α-particle number, a concept much used for neutron pairing in super-fluid states in nuclei [1,32]. For the two-neutron pairing states in heavy nuclei, the transfer of neutron pairs between super-fluid nuclei [32,33] is strongly enhanced. The analogy to the enhancement of transfer of correlated neutron pairs [32,33] is the multiple emission of α-particles as a collective transition (changing the particle number as a collective variable) between nuclei with different numbers of α-particles, from compound nuclei with super-fluid properties. In our case the change of the α-particle number of the condensate must be considered as a strongly enhanced collective transition between the collective α-condensate states, a feature discussed for α-particle transfer between very heavy nuclei in the valley of stability in ref. [34]. Thus the observation of multiple emission of α-particles from the compound state with the mentioned coherent properties can be proposed as the signature for the observation of the collective Bose gas. More specifically, the emission should be strongly enhanced, relative to the statistical model prediction, in the latter case the emission of many α-particles would be observed into different angles [13]. The coherent emission should occur into the same (identical) angle. This will lead to the observation of unbound resonances such as $^8\mathrm{Be}(0^+, 2^+)$ and the excited states of ^{12}C, namely $^{12}\mathrm{C}^\star(0_2^+, 0_3^+)$-clusters. Such a feature may in fact have been observed in the recent data of refs. [12,13].

Another decay mode, which must be mentioned here is the possible decay of the heavier $N = Z$ nuclei by Coulomb explosion. This process is observed in highly charged van der Waals clusters as discussed by Last and Jortner [35]. In our case the simultaneous emission of α-particles is expected with characteristics very different from standard compound-nucleus decay.

5 Discussion and conclusions

Here some final remarks to summarise the proposed scenario of clustering in $N = Z$ nuclei are given. First, we emphasise that only nuclei with $N = Z$ are considered. In the ground states the correlations between four nucleons give rise to some clustering related to α-particles, in such a way that a liquid-drop model can be used in an approximate way for the description of the overall dependence of binding energies on mass number expressed as the number of α-particles.

Based on the success of various models like the Brink-Bloch model [17–19] we have used the concept that α-clusters do exist with some probability inside particular nuclear states in light nuclei and, although with a strongly reduced probability, also in heavier nuclei. They form a mixture of nucleons and α-particles with a smaller chemical potential for the nucleons in the latter. In some exotic nuclei and at higher excitation energies the binding energy per nucleon approaches the value in the α-cluster. A thermodynamic phase transition from fermions to a gas of free α-particles is proposed here, the transition is expected to occur with increasing temperature at excitation energies reaching the cluster thresholds (as proposed in the Ikeda diagram). This transition is very different from those discussed (and observed) due to the pairing interaction between nucleons in nuclei [1] which occurs with decreasing temperature (decreasing excitation energies).

At the critical excitation energy for the condensation into "free" α-particles (strongly interacting via the cited resonances), these represent a Bose gas with a very large de Broglie wavelength of relative motion (15–40 fm). This value is much larger than the distance between two α-particles and covers the whole volume of the nucleus. This may give the gas the properties of a Bose-Einstein condensate as discussed by P. Schuck and collaborators [27,28]. In their work the Bose gas of interacting α-particles also appears at the thresholds for α-particle decomposition and forms nuclear states with a much larger radial extension; the volume of the nucleus may be easily increased by a factor two. However, the Coulomb interaction limits the stability of such systems to mass 40, as discussed in ref. [28]. However, the decay of such states, formed in nuclear reactions in heavier compound nuclei which are unbound, may be observed via the correlated emission of several α-particles forming excited resonant states which form a dilute α-gas, like the ^{8}Be$(0^+, 2^+)$ and the unbound states like the ^{12}C$^\star(0_2^+, 0_3^+)$-clusters. Their emission probability must be compared with that of normal nuclear states.

There remain many open theoretical problems in the study of the α-particle gas in nuclei. The experimental tools for such studies are available, the work by Kokalova *et al.* [12,13] shows that the observable effects are large.

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